Laplace Transfer Function Simulation With SPICE

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OVERVIEW

The design phase of a complex system may include the definition of a Laplace transfer function, in order to test the design for stability.
However, the Laplace transform assumes a continuously linear system, which gives no visibility of effects occurring as a result of limiting due, possibly, to outputs exceeding the power supply voltages.

We present a method of expressing a Laplace transfer function in terms of SPICE simulation primitives, such that the effects of system non-linearity can be included in the simulation by the addition of, for example, clamping diodes.

A transfer function is defined as:

H(s) = Y(s) / U(s)

where Y(s) is the Laplace transform of the output, and U(s) is the Laplace transform of the input. The form of both numerator and denominator is a polynomial expression, where the numerator can be represented by:

B0.s^m + B1.s^m-1 + B2.s^m-2.....+ Bm-1.s + Bm

while the denominator looks like:

s^n + A1.s^n-1 + A2.s^n-2........+ An-1.s + An

For a realisable system, the order of the numerator will be less than or equal to, the order of the denominator.

For any given transfer function, both numerator and denominator are first multiplied by s^-n times an arbitrary function, E(s), (equivalent to dividing by the highest power of s times an arbitrary function).

Y(s) then becomes:

E(s).B0.s^m-n + E(s).B1.s^m-n-1 + E(s).B2.s^m-n-2
.....+ E(s).Bm-1.s^1-n + E(s).Bm.s-n

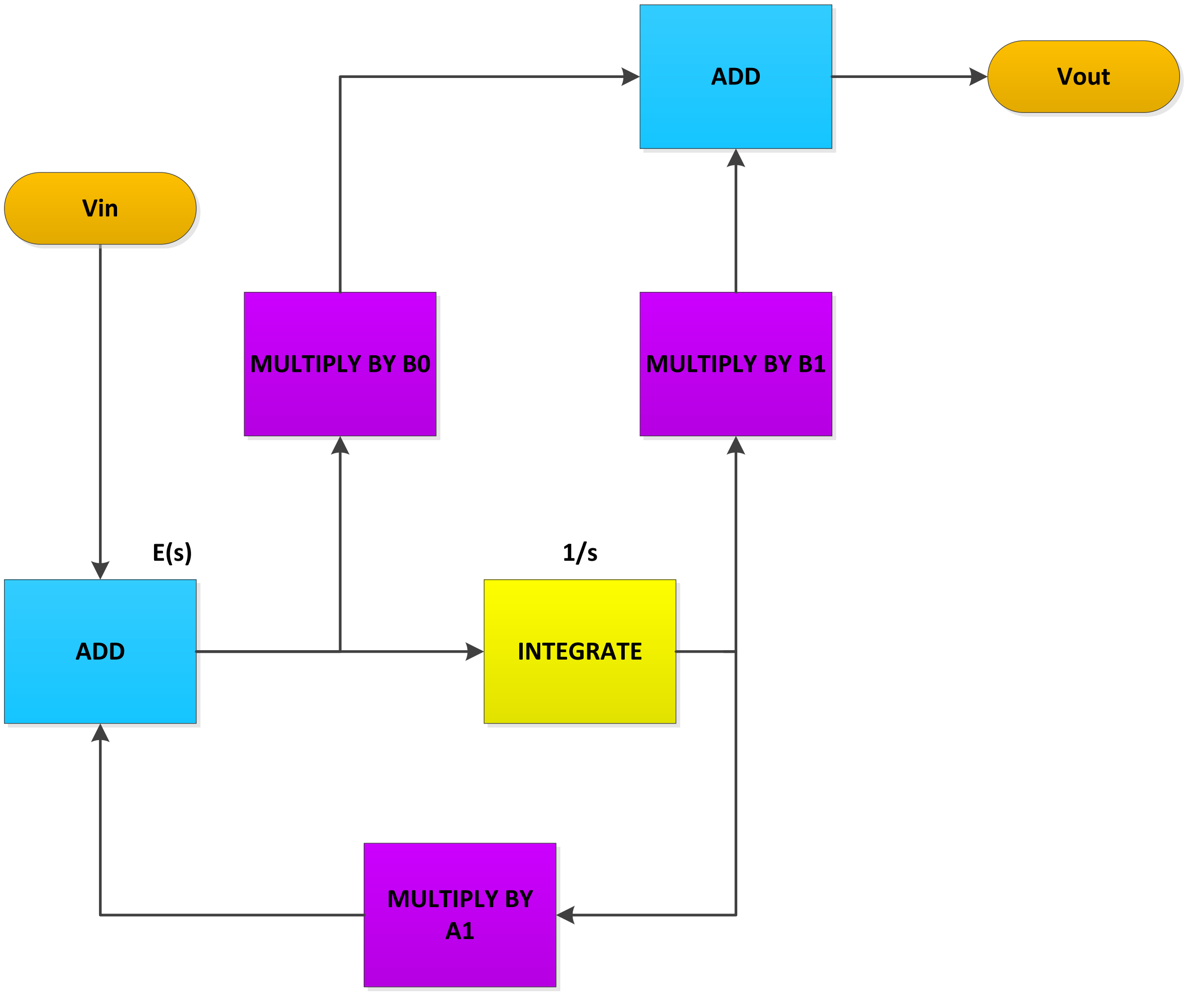
and U(s) becomes:

E(s) + E(s).A1.s^-1 + E(s).A2.s^-2........
+ E(s).An-1.s^1-n + E(s).An.s^-n

which may be written:

E(s) = U(s) - E(s).A1.s^-1 - E(s).A2.s^-2....
- E(s).An-1.s^1-n - E(s).An.s^-n

Using only integrator (for 1/s), adder and constant-multiplier elements, it is possible to realize this type of function within SPICE, using summation as the arbitrary function, E(s).



Transfer Function Element

The above diagram is one n'th of the number needed for a given denominator polynomial, and may be cascaded from Vout to Vin, with the input adder taking in an extra term from the succeeding stage, and the output adder being placed after the last stage.

E(s) appears as an apparent feedback error signal, while the ADD at the top right implements Y(s). The integrator (1/s) function serves to provide the "n" integrations, to form all of the E(s) terms in the above equations, where "n" is the order of the denominator polynomial.

The terms B0, B1, A1, etc are the coefficients of the polynomial, applied through constant-gain multipliers. Provision is made for the same order in the numerator as the denominator, so the unused coefficients may merely be set to zero.

The above block of functions is made generic by applying the coefficients as dc voltages, and grounding any unused coefficient inputs. Thus, one block may be cascaded "n" times, and "m" coefficients may be defined, to simulate any transfer function.

Chebyshev Filter example

The transfer function of the filter is:

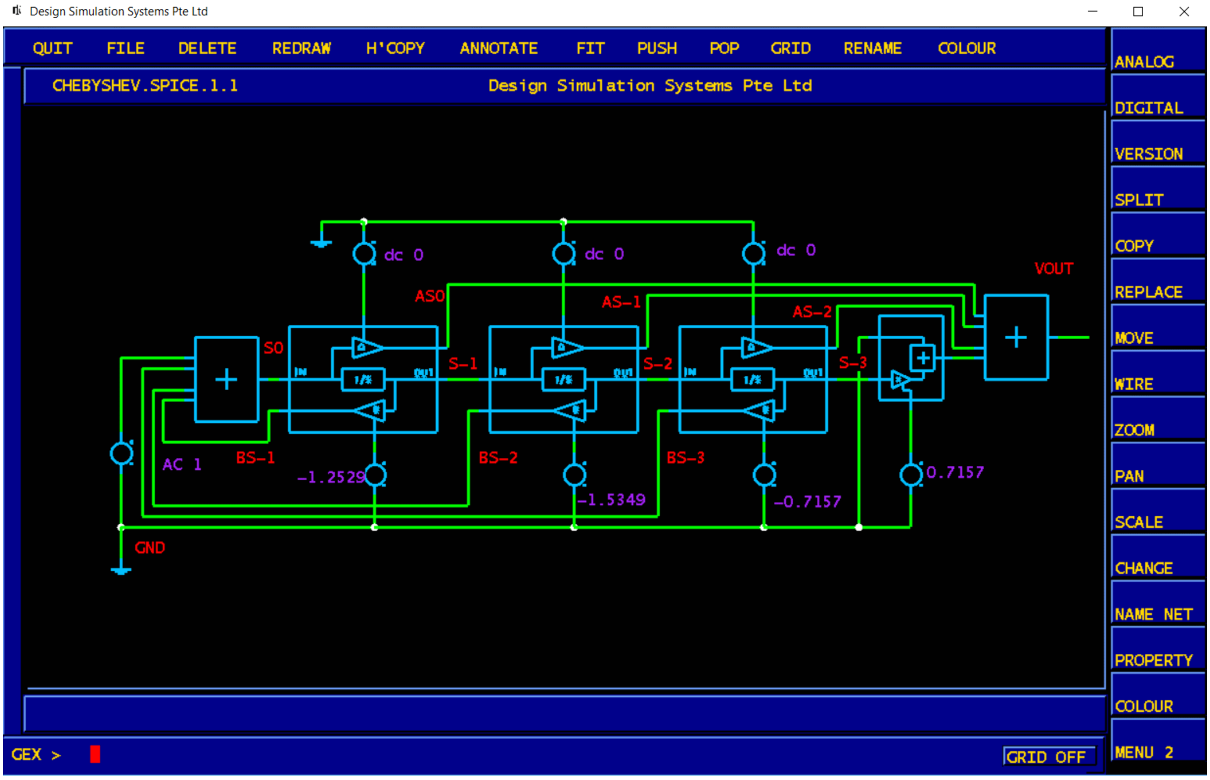
$$(0.7157) / (s^3 + 1.2529s^2 + 1.5349s + 0.7157)$$

Dividing top and bottom by s^3 gives:

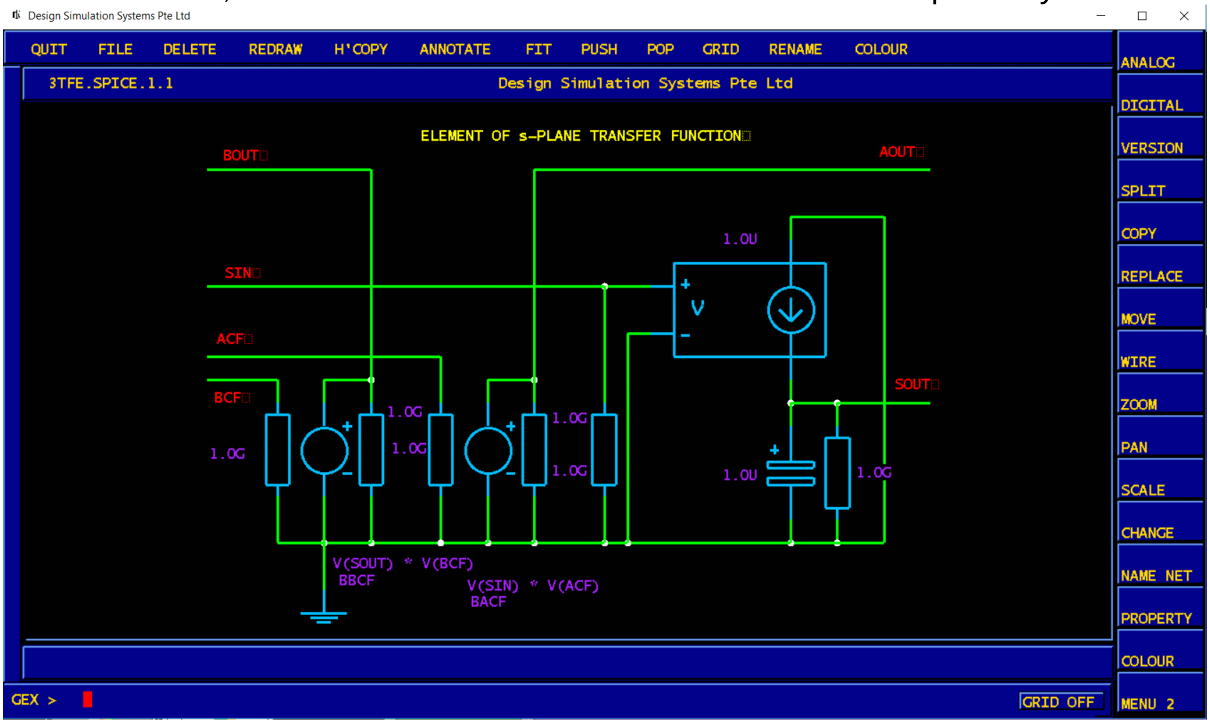
$$(0.7157s^{-3}) / (1 + 1.2529s^{-1} + 1.5349s-2 + 0.7157s^{-3})$$

Since the numerator is a pure number, we will need a weighted adder at the output but, since there are no other numerator terms, all forward coefficients may be either grounded, or set to DC 0 volts.

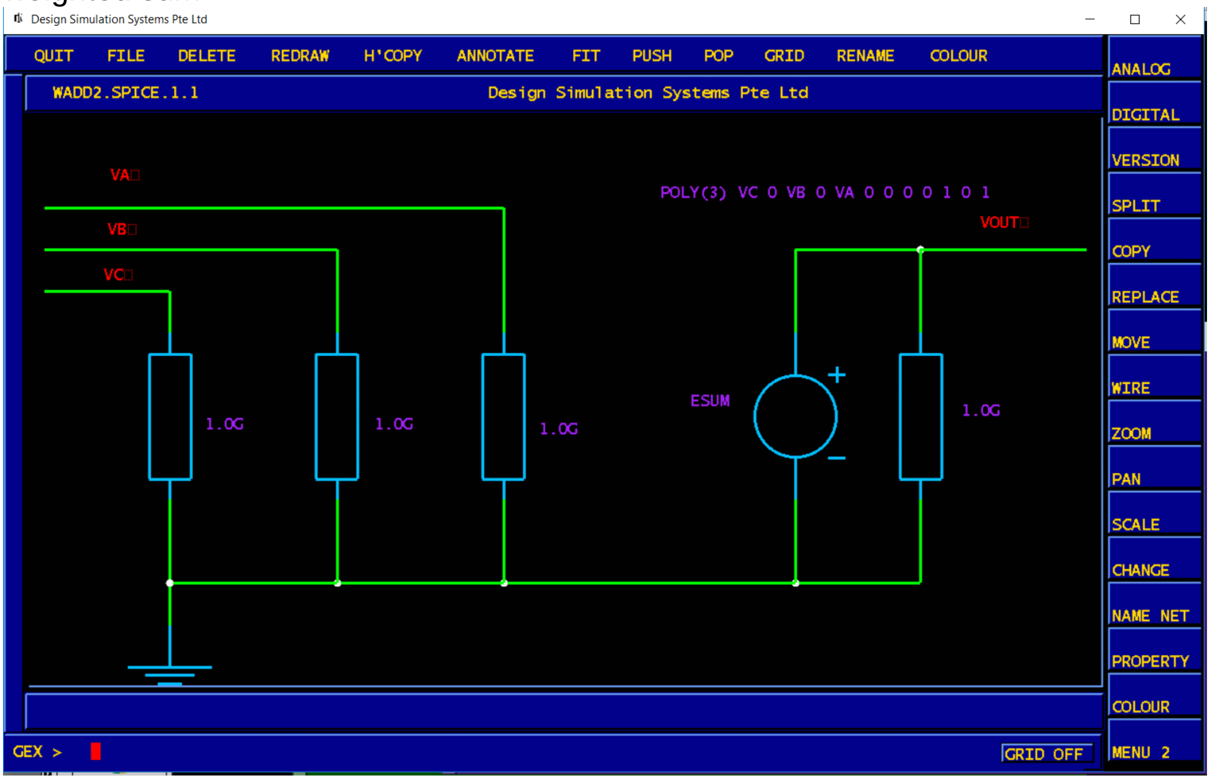
Since there are three terms in the denominator, three cascaded transfer function elements will be required, to supply s^{-1}, s^{-2} and s^{-3}, and the three voltage sources defining the coefficients will have values of (-1.2529v), (-1.5349v) and (-0.7157v). The weighting function of the output from the last stage will be (0.7157) and we will need one four-input adder at the input. For reasons of simplicity, the topology has been left constant, and there is a second adder at the output, but all its inputs are zero.



All of the elements discussed so far are provided in the library "laplace".
The Transfer Function Element block is designed in two forms, for use with Spice3 and SPICE2G6, and the blocks are entitled "3TFE" and "TFE" respectively.

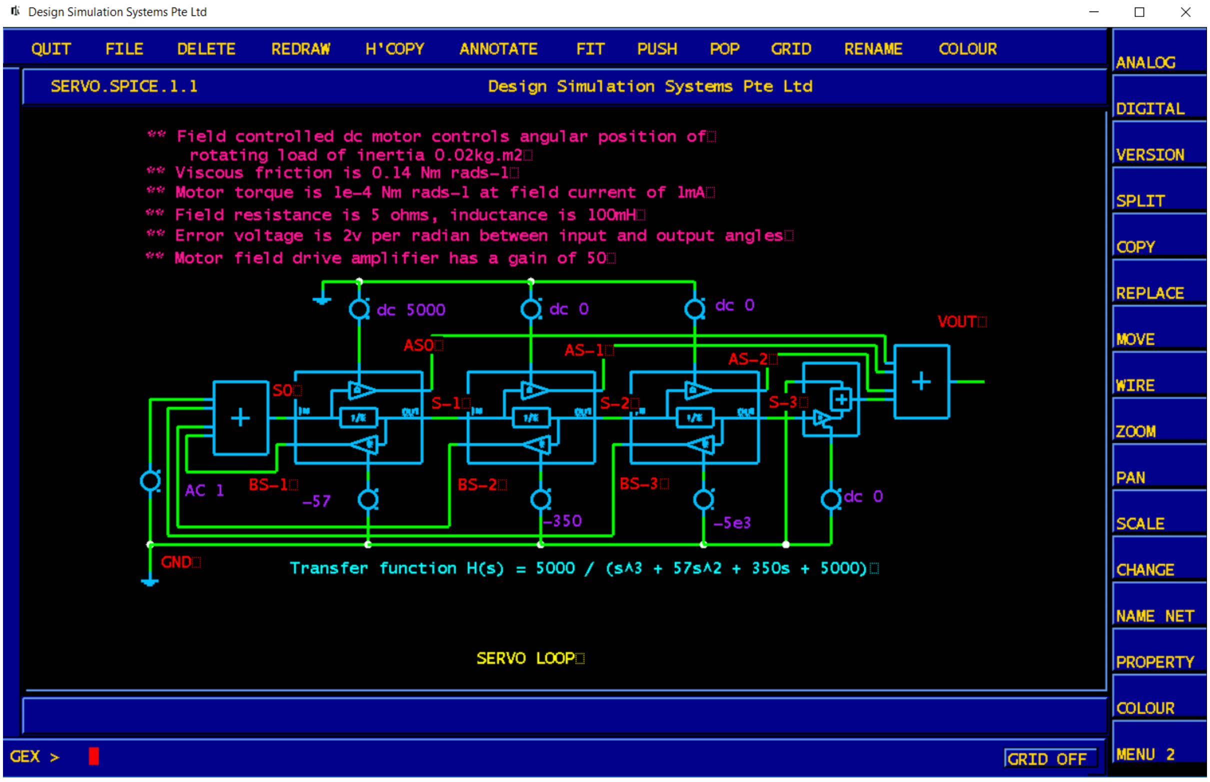


Multi-input adders are provided, with the same naming convention, i.e, a 5 input adder for use with Spice3, is called "3ADD5" etc. Additionally, to cater for gain from the high-order stage, an adder with one weighted input is supplied, named "3WADD2" or "WADD2", which may be cascaded with any other adder to form a weighted sum.



In addition to the Chebyshev filter , the following additional examples have been included to make the principle clear, and to provide a template for future designs:

- **servo** A complete servo system, including motor



- **tunedcct** A tuned circuit
- **hpf** High-pass filter
- **control** Multiple loop control system, on which all of the above are based.

Conclusion

A Laplace transfer function can be easily expressed in terms of SPICE primitives, which are so simple, that the simulation of complex systems is extremely fast. Additionally, non-linear systems may be simulated by the addition of simple SPICE limiter circuits.